Optimal Control Theory

Chapter one:-

1) Laplace Transform
2) Inverse laplace Transform
3) Special Functions
4) Solving Initial value problem

اهم الماخوذ منها :-

1) introduction Contoral theory including optimal control theorey

Example solve the following I.v.p laplace transform via laplace transform

$$\ddot{x}+4x=2u_s(t) \qquad x(0)=0 \quad , \dot{x}(0)=0$$

Sol $\mathcal{L}[\ddot{x}+4x]=\mathcal{L}\, 2\overset{=1}{u_s(t)}$

$$\Rightarrow \mathcal{L}[\ddot{x}]+4\mathcal{L}[x]=\frac{2}{s} \Rightarrow s^2X(s)-sX(0)-\dot{x}(0)+4X(s)=\frac{2}{s}$$

$$s^2X(s)+4X(s)=\frac{2}{s}$$

$$X(s)(s^2+4)=\frac{2}{s} \Rightarrow X(s)=\frac{2}{s}\cdot\frac{1}{s^2+4}$$

Theorem:- Shift on S-axis

$$\mathcal{L}\{e^{-\alpha t}f(t)\}=F(s+\alpha)$$

Theorem :-

$$\mathcal{L}\{t\,f(t)\}=-\frac{d}{ds}F(s)$$

And in general for $n=1,2,\ldots$

$$\mathcal{L}\{t^n f(t)\}=(-1)^n\frac{d^n}{ds^n}F(s)$$

Theorem :- (Convolution theorem) -

$$f(t)=\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\{G(s)H(s)\}=(g*h)(t)=(h*g)(t)$$ read "Convolution of g and h"

Example Determine the inverse laplace transform

$$F(s)=\frac{1}{s}\cdot\frac{1}{s+1}$$

Sol $G(s)=\frac{1}{s} \Rightarrow g(t)=1$

$$H(s) = \frac{1}{s+1} \Rightarrow h(t) = e^{-t}$$

$$f(t) = \mathcal{L}^{-1} F(s)$$

$$f(t) = (g * h)(t) = \int_0^t g(t-\tau)\, h(\tau)\, d\tau = \int_0^t e^{-\tau}\, d\tau$$

$$f(t) = (h * g)(t) = \int_0^t e^{-(t-\tau)}\, d\tau$$

Chapter one

laplace Transformation in system analysis, one often encounters time-invariant linear differential equations of second order or higher order. and it is generally difficult to obtian solutions to these equations in closed form via the solution methods in ordinary differential equations. one way to circumvent this difficulty, however, is to use laplace transformation.

The laplace Transformation is defined by:-

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt \qquad (1.1)$$

Theorem 1:- Existence of laplace transform Given that Conditions:-

C1) $f(t)$ is piecwise Continuous on every finite (time) interval of $[0, \infty)$

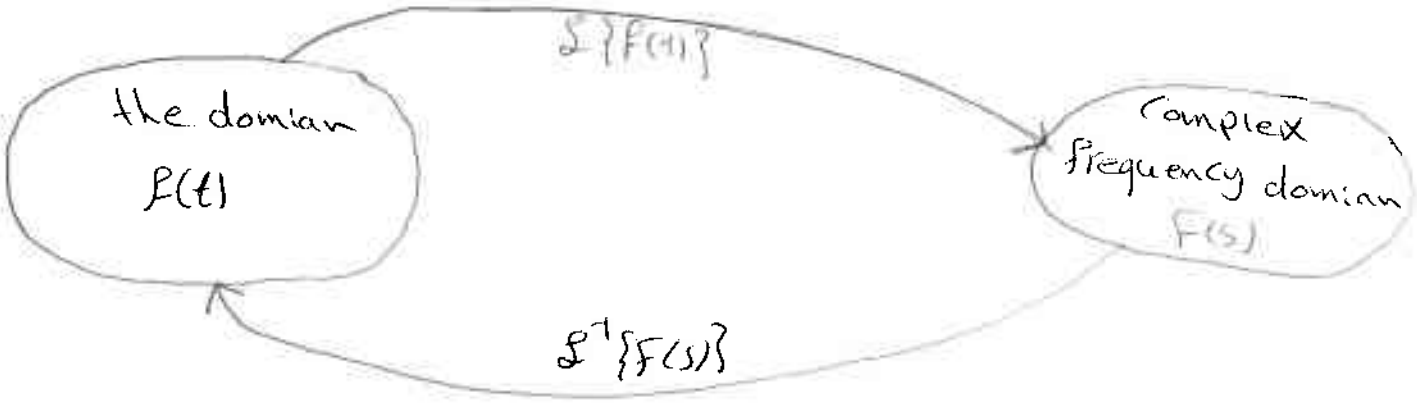
C2) the magnitude of $f(t)$ is bounded by an exponinatial function :- $|(f(t))| \leq m e^{at}$

for some real constants a and m and $\forall t \in [0, \infty)$ hold for $f(t)$ then its laplace transform, $F(s)$ exists for $\forall Re\{s\} > a$

Inverse laplace Transform:-

The inverse laplace transform is represented by:-

$$f(t) = \mathcal{L}^{-1}\{F(s)\} \quad -- \quad (1.2)$$



Special Functions:-

1) Unite-step Function $U_s(t)$, $U(t)$

$U_s(t) = 1$

2) Unite-ramp Function $U_r(t)$

$U_r(s) = t$

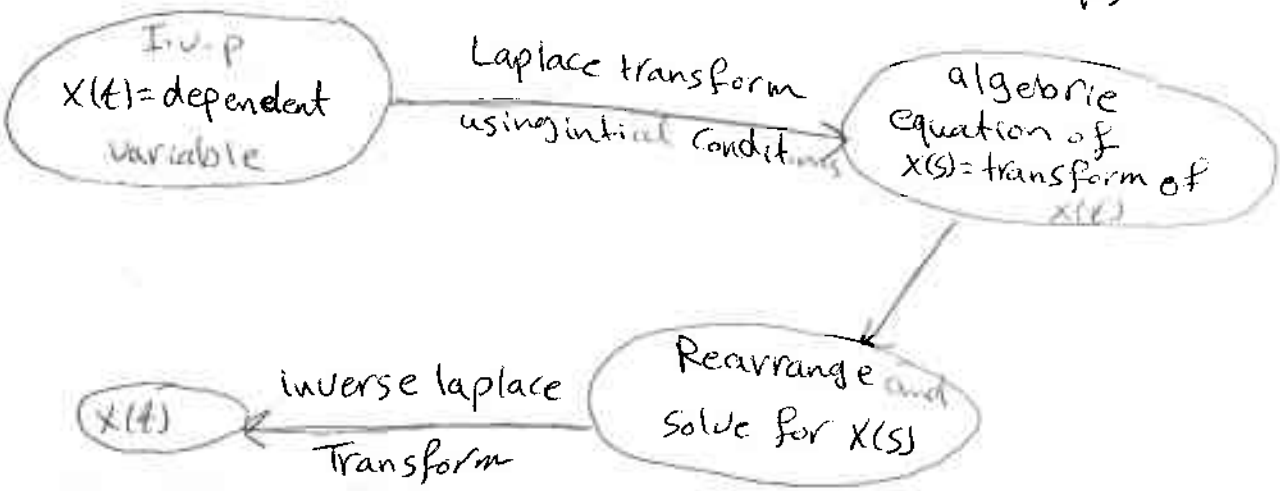
3) Unite-pulse Function $U_p(t)$

$$U_p(t) = \begin{cases} 1 & 0 < t < t_1 \\ 0 & t < 0 \ \& \ t > t_1 \end{cases}$$

4) Unite-sinusoidal function

$$f(t) = \begin{cases} \sin \omega t & t \geq 0 \\ 0 & t < 0 \end{cases} \quad \Bigg| \quad g(t) = \begin{cases} \cos \omega t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Exponential function

$$f(t) = \begin{cases} e^{-at} & t > 0 \\ 0 & t < 0 \end{cases}$$

Solving Initial value problem (I.V.P)